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This paper two

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

~~The research reported~~ falls under ~~the~~ headings: ~~9.(a)~~ Order Statistics and ~~9.(b)~~ Nonparametric Statistics.

~~9.(a)~~ Topics investigated include the following: robustness and estimation in the presence of outliers; inequalities for order statistics and functions of order statistics; moving order statistics.

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20. (continued)

(b) The second edition of a research monograph The Method of Paired Comparisons was published. New nonparametric methods of analyzing unbalanced paired-comparison experiments were developed. Other topics: rank statistics and U-statistics; censored data; extreme-value theory.

ORDER STATISTICS AND ROBUST INFERENCE

FINAL REPORT

H. A. DAVID

NOVEMBER 30, 1988

U. S. ARMY RESEARCH OFFICE

CONTRACT
DAAG29-85-K-0189

IOWA STATE UNIVERSITY

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1. FOREWORD

This final report covers the period July 1, 1985 to September 30, 1988, the extra three months being due to an extension granted in 1987. Six semi-annual progress reports have been issued, covering the first three years.

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2. STATEMENT OF PROBLEMS STUDIED

Two general areas were investigated: A. Order Statistics and B. Nonparametric Statistics.

The PI and his students concentrated on the following problems.

A. Relatively little is known about the finite-sample behavior of order statistics except in random sampling. One deviation from random sampling of particular importance is the presence of outlying observations. A major aim of this research was to assess and bound the effects of the outliers for linear functions of order statistics such as trimmed means. Also of interest is the behavior of order statistics under various assumptions of positive or negative dependence among the observations. Moving medians, and more generally, moving linear functions of order statistics, introduced initially in quality control and later as useful descriptive statistics, have recently found wide use by electrical engineers concerned with digital filtering. The behavior of these statistics under non-standard conditions such as a trend in the observations is therefore of considerable additional interest.

B. In a balanced paired-comparison experiment every object is compared equally often with every other object, the results of the experiment being a set of 0-1 (or possibly more elaborate) scores, representing the outcome of each comparison. The analysis of such experiments is well understood. However, in practice there is often a lack of balance. Although methods exist for handling such data when strong model assumptions are made, it is important to be able to deal with the frequently occurring case when no such assumptions are warranted.

Various short-term visitors have considerably expanded the scope of the above investigations, as noted in the next section.

3. SUMMARY OF RESEARCH FINDINGS

A. Order Statistics [2, 3, 5, 6, 9, 11, 12, 16, 19, 20, 22, 24]

Motivated largely by the need for procedures that are robust in the presence of outliers, a new theme has been the study and development of inequalities for order statistics [3, 6, 20, 24]. In [6] a unified approach provides upper bounds for functions of order statistics such as are used in tests for outliers. For L-estimators (linear functions of order statistics) a method of bounding the bias caused by the presence of outliers is given in [3]. Estimation in the presence of outliers is treated in [11]. The behavior of order statistics under various dependence assumptions on the observations has been investigated in a recently completed Ph.D. thesis by S. H. Kim

The study of moving medians, moving ranges, and moving L-statistics has in recent years become a very active field in the engineering literature because of its relevance to digital filtering. David (Biometrika, 1955) was an early paper in this area; a much more complete treatment is given by David and Rogers (Biometrika, 1983). A review, with some additional results in the presence of trend, constitutes part of [24].

Paper [16] is the final contribution in an extensive study of the selection differential made by H. N. Nagaraja, a former Ph.D. student.

Articles [2] and [5] are entries in the Encyclopedia of Statistical Sciences.

B. Nonparametric Statistics [1, 4, 7, 8, 10, 13, 14, 15, 17, 18, 21, 23]

The monograph [1] is a substantial revision of the first edition published in 1963 (with ARO support). Incorporated *inter alia* is a description of the simple new nonparametric method developed in [4] for handling unbalanced paired-comparison data. A further study of the properties of this method has since been made. Also treated are methods for the selection of the best object, including the closed adaptive sequential procedure proposed in [8].

Other research in nonparametric statistics has been carried out by a series of visitors (and their collaborators). Noteworthy are M. Ghosh's papers [13, 14, 15] on U-statistics and rank statistics. Aspects of the important topic of how to handle censored data are tackled in [21] and [23]. Nonparametric extreme-value results are obtained in [10].

4. SCIENTIFIC PERSONNEL SUPPORTED

*D. M. Andrews, Research Assistant	M.S. (1986)
H. A. David, Principal Investigator	
M. Ghosh, Visiting Professor	
I. H. Ha, Research Assistant	M.S. (1985)
B. K. Kale, Visiting Professor	
S. H. Kim, Research Assistant	Ph.D. (1988)
*J. Liu, Research Assistant	M.S. (1988)
M. P. Rogers, Research Assistant	Ph.D. (1985)
Tiago de Oliveira, Visiting Professor	
K. J. Wang, Research Assistant	M.S. (1987)
S. S. Yang, Visiting Associate Professor	

*Current Ph.D. students

5. LIST OF PUBLICATIONS

Book Published

1. David, H. A. (1988). The Method of Paired Comparisons, Second Edition. Griffin, London; Oxford University Press, New York. 188 pages.

Papers Published

2. David, H. A. (1985). Order Statistics. In Encyclopedia of Statistical Sciences, Vol. 6, pp. 504-510.

3. David, H. A. (1986). Inequalities for Ordered Sums. Annals of the Institute of Statistical Mathematics, **38** 551-555.

4. David, H. A. (1987). Ranking from Unbalanced Paired-Comparison Data. Biometrika, **74** 432-436.

5. David, H. A. (1988). Studentized Range. Encyclopedia of Statistical Sciences, Vol. 9, pp. 39-43.

6. David, H. A. (1988). General Bounds and Inequalities in Order Statistics. Special issue on "Order Statistics and Applications" of Communications in Statistics - Theory and Methods, **17** 2119-2134.

7. David, H. A. (1988). Note on "An Extreme Null Distribution Approach to the Problem of Paired Comparisons" by Patricia S. Costello and Douglas A. Wolfe. Communications in Statistics - Theory and Methods, **17** 4005-4009.

8. David, H. A. and D. M. Andrews. (1987). Closed Adaptive Sequential Paired-Comparison Selection Procedures. Journal of Statistical Computation and Simulation, **27** 127-141.

9. David, H. A. and I.-H. Ha. (1985). Replacement of Parents by Meritorious Offspring in Within-Family Selection. Biometrics, **41** 1045-1048.

10. de Haan, L. and I. Weissman. (1988). The Index of the Outstanding Observation Among n Independent Ones. Stochastic Processes and their Applications, **27** 317-329.

11. Gather, U. and B. K. Kale. (1988). Maximum Likelihood Estimation in the Presence of Outliers. Communications in Statistics - Theory and Methods, **17** 3767-3784.

12. Ghosh, J. K. and C. K. Mustafi. (1986). A Note on the Residual Median Process. Canadian Journal of Statistics **14** 251-255.

13. Ghosh, M. (1984). Rank Statistics and Limit Theorems. In Handbook of Statistics, Vol. 4, pp. 145-171.

14. Ghosh, M. (1985). Berry-Esseen Bounds for Functionals of U-Statistics. Sankhya, A, **47** 255-270.

15. Ghosh, M. and Sen, P. K. (1984). On Asymptotically Risk-Efficient Sequential Versions of Generalized U-Statistics. Sequential Analysis, **3**, 231-249.

16. Nagaraja, H. N. (1984). Some Nondegenerate Limit Laws for Sample Selection Differential and Selection Differential. Sankhya, A, **46** 355-369.

17. Stephenson, W. R. and M. Ghosh. (1985). A General Class of One-Sample Nonparametric Test Statistics Based on Subsamples. Communications in Statistics - Theory and Methods, **14**, 1669-1684.

Papers Accepted for Publication

18. David, H. A. Ranking and Selection from Paired-Comparison Data. To appear in the Proceedings Volume of the 2nd International Conference on Inference Procedures Associated with Statistical Ranking and Selection.

19. Gather, U. and B. K. Kale. Outlier Generating Models -- A Review. To appear in Festschrift for Dr. Nagabhushanam, C. G. Khatri, Ed.

20. Liu, J. and H. A. David. Quantiles of Sums and Expected Values of Ordered Sums. To appear in Australian Journal of Statistics.

21. Meeden, G., M. Ghosh, C. Srinivasan, and S. Vardeman. The Admissibility of the Kaplan-Meier and Other Maximum Likelihood Estimators in the Presence of Censoring. To appear in Annals of Statistics.

Papers Submitted for Publication

22. Tiago de Oliveira, J. Intrinsic Estimation of the Dependence Structure for Bivariate Extremes. Submitted to Statistics and Probability Letters.

23. Yang, S.-S. A Nonparametric Procedure for Comparing Two Survival Distributions Based on Randomly Right Censored Data. Submitted to Journal of the American Statistical Association.

Also

24. David, H. A. Some Applications of Order Statistics. Keynote Address. To appear in the Proceedings of the 34th Conference on the Design of Experiments in Army Research, Development and Testing, new Mexico State University, Las Cruces, October 19, 1988.